EFFECT OF A SHOCK WAVE ON HEAT PROPAGATION

L. P. Gorbachev and V. F. Fedorov

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The nature of the propagation of a thermal wave produced by a powerful explosion was described in a number of papers, for example, [1-6]. It was shown by a numerical method [4] that a shock wave is present together with the thermal wave. In this paper, the effect of a homothermal shock wave on heat propagation is evaluated by an approximate method.

At some initial time let an energy $E_0$ be deposited and let spherical thermal waves and spherical shock waves be propagated from the point of energy deposition. We consider a time interval such that one can neglect radiative energy and need not consider the formation of an external shock wave at the thermal front. The gas is assumed ideal, and the physical properties of the heated air are taken into account by the introduction of effective values for the adiabatic index $\gamma$ and the molecular weight $\mu$.

The equation of heat transport for a thermal wave including radiation is written in the radiative thermal-conductivity approximation as

$$\frac{R\rho}{\mu(\gamma-1)}\frac{dT}{dt}-\frac{RT}{\mu}\frac{d\rho}{dt}=-\operatorname{div}S, \tag{1}$$

where S is the flux of radiant energy.

At high temperature, one can roughly assume the temperature to be constant over the entire heated region:

$$\begin{cases} T(r,\ t)=T(t),\ r\leqslant r_T; \\ T(r,\ t)=0,\ r>r_T, \end{cases}$$

where $S_T$ is the radius of the thermal-wave front.

On the shock front ($r = r_1 < r_T$) being propagated in the heated gas, relations must be satisfied which reflect conservation laws at the isothermal compression discontinuity:

$$\begin{cases} \rho_1(D-v_1)=\rho_0 D; \\ p_1+\rho_1(D-v_1)^2=p_0+\rho_0 D^2; \\ \frac{(D-v_1)^2}{2}+\frac{S_0-S_1}{\rho_0 D}=\frac{D^2}{2}, \end{cases} \tag{2}$$

where $S_0 - S_1$ is the radiation flux from the shock front. We assume that it results from motion of the gas.

We have from Eqs. (2)

$$S_0-S_1=\frac{\rho_0 D^3}{2}\left(1-\frac{a^4}{D^4}\right), \tag{3}$$

where D is the velocity of the shock front and $a = \sqrt{RT/\mu}$ is the isothermal velocity of sound.

Because of its motion, the gas possesses kinetic energy along with internal energy, with the total energy $E_0$ being conserved,

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$$E_0 = E_T + E_K \quad \text{or} \tag{4}$$

$$4\pi\left[\int_0^{r_T} \frac{\rho RT}{\mu(\gamma-1)} r^2 dr + \int_0^{r_1} \frac{\rho v^2}{2} r^2 dr\right] = E_0.$$

Propagation of the thermal wave depends on the density and velocity distribution of the gas behind the shock front.

One can roughly assume that

$$\rho = \rho_1\left(\frac{r}{r_1}\right)^m, \quad m = 3\left(\frac{\rho_1}{\rho_0} - 1\right); \tag{5}$$

$$v = D\left(1 - \frac{\rho_0}{\rho_1}\right)\frac{r}{r_1}. \tag{6}$$

Equation (5) was proposed by Ya. B. Zel'dovich and is valid for a strong shock [7]. The problem is solved approximately. Multiplying Eq. (1) by $4\pi r^2$, integrating from 0 to $r_T$, and taking Eqs. (3)-(6) into consideration, we obtain

$$\frac{M_T R}{\mu(\gamma-1)}\frac{dT}{dt} = 4\pi r_1^2 \rho_0 D^3\left[\frac{(1 - a^4/D^4)}{2} - \frac{(1 - a^2/D^2)^3}{3 + m}\right] - 4\pi r_T^2 S_T, \tag{7}$$

where $S_T$ is the radiation flux from the thermal-wave front; $M_T = 4\pi\rho_0 r_T^3/3$. With the equation of motion taken into consideration, Eq. (1) reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial t}\left(\frac{\rho RT}{\mu(\gamma-1)} + \frac{\rho v^2}{2}\right) = -\frac{1}{r^2}\frac{\partial}{\partial r} r^2 v\left(\frac{\rho RT}{\mu(\gamma-1)} + \frac{\rho v^2}{2} + p\right) - \frac{1}{r^2}\frac{\partial}{\partial^2}(r^2 S). \tag{8}$$

Integrating Eq. (8) within the boundaries of the region bounded by the shock wave, we obtain

$$\begin{cases} \dfrac{dE_K}{dt} + \dfrac{M_1 R}{\mu(\gamma-1)}\dfrac{dT}{dt} = -4\pi r_1^2 S_0; \\ \dfrac{M_1 R}{\mu(\gamma-1)}\dfrac{dT}{dt} = -4\pi r_1^2\left[S_1 + \dfrac{3}{m+3}\rho_0 D^3 (1 - a^2/D^2)\right], \end{cases} \tag{9}$$

where $M_1 = 4\pi\rho_0 r_1^3/3$.

Eliminating the gas temperature from Eqs. (9) and using Eqs. (3) and (4), we obtain an equation which describes the variation of the internal energy $E_T$ of the thermal wave,

$$\frac{dE_T}{dt} = 4\pi r_1^2 \rho_0 D^3\left[\frac{(1 - a^4/D^4)}{2} - \frac{3(1 - a^2/D^2)}{m+3}\right]. \tag{10}$$

Substituting the value

$$E_T = \frac{4\pi}{3}\rho_0 r_T^3 \frac{RT}{\mu(\gamma-1)}$$

in Eq. (10) and using Eq. (7), we have

$$\frac{dr_T}{dt} = \frac{S_T \mu(\gamma-1)}{\rho_0 RT}. \tag{11}$$

To determine the propagation law for the thermal-wave front in accordance with Eq. (10), it is necessary to specify the radiation flux $S_T$ from the front.

If the temperature in the thermal wave is kept strictly constant along the radius, the radiation flux is then

$$S_T = \sigma T^4. \tag{12}$$

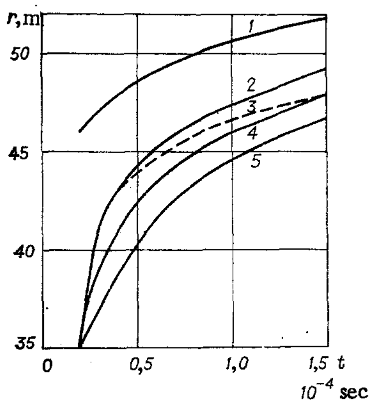


Fig. 1

As a matter of fact, the temperature varies along r, which leads to a change in the quantity (12) by a factor $\xi(l_R/r_T)$ where $l_R$ is the Rosseland range.

We choose the numerical coefficient $\xi$ so that the law for propagation of the thermal-wave front without inclusion of gas motion agrees with that known from the self-similar solution [1].

If one assumes $l_R = (T_k^0/10^6)^2$ m, then

$$S_T = \frac{7.52}{r_T}\left(\frac{T}{10^6}\right)^2 \sigma T^4. \tag{13}$$

We determine the propagation of the shock wave using Eqs. (4)-(6) and the relation (2) at the shock front,

$$\frac{dr_1}{dt} = D = \left[\frac{3(E_0 - E_T)}{2\pi r_1^3 \rho_0}\left(\frac{m+5}{m+3}\right)\right]^{1/2}\left(\frac{1}{1-a^2/D^2}\right). \tag{14}$$

The gas temperature is calculated from

$$T = \frac{3E_T(\gamma-1)\mu}{4\pi\rho_0 R r_T^3}. \tag{15}$$

For a strong shock wave, where the conditions $a^2/D^2 \ll 1$ and $m \gg 5$ are satisfied, we have from Eqs. (10)-(14)

$$\begin{cases} \dfrac{dE_T}{dt} = -\dfrac{dE_K}{dt} = 2\pi r_1^2 \rho_0 D^3; \\ \dfrac{dr_1}{dt} = D = \left(\dfrac{3E_K}{2\pi r_1^3 \rho_0}\right)^{1/2}; \\ \dfrac{dr_T}{dt} = \dfrac{\mu(\gamma-1)\,7.52\cdot 10^{-12}\sigma}{\rho_0 R}\left(\dfrac{3\mu(\gamma-1)}{4\pi\rho_0 R}\right)^5 \dfrac{E_T^5}{r_T^{16}}. \end{cases} \tag{16}$$

We have from the first two equations in (16)

$$\frac{1}{3}\frac{d}{dt}\left(D^2 r_1^3\right) = -r_1^2 D^3.$$

Substituting $d/dt = D(d/dr_1)$ and integrating, we obtain

$$r_1 = c_1(t + c_2)^{1/4}, \tag{17}$$

where $c_1$ and $c_2$ are constants to be determined.

Using Eq. (17), the values of $E_T$ and $r_T$ are determined from Eq. (16):

$$\begin{aligned} E_T &= E_0(1 - e\tau^{-3/4}); \\ r_T &= \xi_0 E_0^{5/17}\{\tau[1 - 20e\tau^{-3/4} - 20(e\tau^{-3/4})^2 + \\ &+ 8(e\tau^{-3/4})^3 - 5/2(e\tau^{-3/4})^4 + 4/11(e\tau^{-3/4})^5] + \\ &c_3\}^{1/17} \simeq \xi_0 E_0^{5/17}[\tau(1 - 20e\tau^{-3/4}) + c_3]^{1/17}, \end{aligned} \tag{18}$$

where $\tau = t + c_2$;

$$e = \frac{4\pi}{3}\frac{\rho_0}{E_0}\frac{c_1^5}{16},$$

$$\xi_0 = \left[\frac{17\mu(\gamma-1)\,7.52\cdot 10^{-12}\sigma}{R\rho_0}\left(\frac{3\mu(\gamma-1)}{4\pi\rho_0 R}\right)^5\right]^{1/17}.$$

The coefficients $c_1$, $c_2$, and $c_3$ are determined by the condition that the radii of the shock and thermal fronts are, respectively, $r_{10}$ and $r_{T0}$ at a given time $t_0$ and a (1-k)-th portion of the explosion energy is transferred to the kinetic energy.

Note that for $\tau \to \infty$, the value of $r_T$ from Eq. (18) transforms into the well-known solution of [1].

The system of ordinary differential equations (10)-(15) was integrated numerically on a computer by the Runge–Kutta method. The following values were used as input data:

$$E_0=4.18\cdot10^{15}\ \text{J};\qquad t_0=2\cdot10^{-5}\ \text{sec};$$
$$r_{T0}=35\ \text{m};\ r_{10}=8\ \text{m};\ \gamma=1.37;\ \mu=2.07;$$
$$E_{T0}=kE_0.$$

The calculations were performed for k = 0.3, 0.5, and 0.7.

The results of numerical calculations of the time dependence of the radius of the thermal front are given in Fig. 1. Curve 1 corresponds to the self-similar solution [1] ($E_T = E_0 = 4.18\cdot10^{15}$ J, k = 1); curves 2, 4, and 5 correspond to the values k = 0.7, 0.5, and 0.3, and curve 3 corresponds qualitatively to the numerical calculations in [4]. Note that for k = 0.7 and 0.5, the numerical solutions agree satisfactorily with the solutions obtained from Eq. (18).

The results indicate that the shock wave has a significant effect on the propagation of heat.

Finally, we evaluate the accuracy of the selected approximate method as exemplified by the propagation of a homothermal shock wave. According to the exact self-similar solution of [3], the radius of the shock front, for a density ratio $\lambda = 2$ at the front, is

$$r_1 = \xi_1 \left(\frac{E_0}{\rho_0}\right)^{1/5} t^{2/5} = \left(\frac{E_0}{\alpha\rho_0}\right)^{1/5} t^{2/5}, \tag{19}$$

where $\alpha = 0.0643 + [0.163/(\gamma - 1)]$. If one assumes $\gamma = 1.37$, $\xi_1 = 1.12$. In the approximate method, the radius $r_1$ is determined from Eq. (19) where

$$\alpha = \frac{16\pi}{75}\left(\frac{3(\lambda-1)^2}{2\lambda(3\lambda+2)} + \frac{\lambda-1}{\lambda^2} - \frac{1}{\gamma-1}\right),$$

if one assumes as in the exact solution that $\lambda = 2$, $\gamma = 1.37$, then $\xi = 1.11$. Note that a value of 1.04 is obtained for $\xi_1$ from the thin-layer method of Chernyi [8].

The comparisons made demonstrate the acceptability of approximate methods for describing the behavior of thermal and shock-front propagation.

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